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Visualizing Heat Transfer for a Second Grade Fluid over a Linear Shrinking Sheet: A Graphical Approach

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Abstract
This current research paper explores the heat transfer phenomenon of a second-grade fluid flowing past a linear shrinking sheet. By employing similarity transformations, the governing set of boundary layer partial differential equations is transformed into a set of non-linear ordinary differential equations. The numerical solution to this problem is obtained using the widely utilized Runge-Kutta method in conjunction with a shooting technique. Detailed graphical representations are presented, showcasing the calculated results for skin friction coefficient, temperature profiles, and local rate of heat transfer, for selected parameter values. Intriguingly, it is observed that under specific parameter conditions, a dual solution exists. Furthermore, an interesting finding emerges, indicating that an increase in the viscoelastic parameter leads to an augmentation in the thickness of both the momentum and thermal boundary layers, irrespective of whether it is an upper or lower branch solution.

Keywords: Second grade fluid, Shrinking sheet, Heat transfer, Numerical solution, Dual solutions.

Introduction
Several technical procedures include boundary layer flow across a stretched surface. Such situations arise in a variety of sectors, including paper manufacturing, polymer processing, glass or fiber sheet manufacture, textile manufacturing, and many more. Initially Sakiadis [1,2] work on flow over a stretching sheet. He discussed boundary layer flow of steady viscous fluid over a linear sheet. Many of researchers doing enhancement their wok and find different valuable results in different conditions regularly. Vijendra Singh and Shweta Agarwal [3]discuss the heat transfer for second grade fluid and second order fluid on a stretching sheet. In their work they take the sheet which is exponentially stretched and, in their observation, the thermal conductivity is taken variable. Some researchers found that flow over a shrinking sheet has also useful applications in many technological processes such as packaging of bottles and food, to make computer and transistors, making high performance wire, in textile industry and many others [4-11].Recently, Van Gorder and Vajruvelu [12] found the dual solution for flow of a second grade fluid past a shrinking sheet with magnetic effect. The author's work is limited to flow of the fluid. Previous studies have not explored the heat transfer phenomenon associated with the flow over a shrinking sheet combined with a linear stretching sheet. A recent investigation conducted by Fatima and Hayder [13] focused on examining the flow and heat transfer characteristics within a multi-mini channel heat sink. Their study specifically investigated the impact of channel configuration on

fluid flow and heat transfer. In contrast, the present study aims to calculate the heat transfer solution for a second-grade fluid flowing over a shrinking sheet.

Problem formulation

Let's examine the characteristics of a steady two-dimensional boundary layer flow involving a second-grade fluid with electrical conductivity over a shrinking sheet positioned in the y = 0 plane. To introduce a magnetic influence in the x-direction, a vertical uniform magnetic field with an intensity of B_0 is imposed along the y-axis. Let us consider that u is the velocity component in x axis and v is the velocity component in its perpendicular direction, σ_t is thethermal conductivity and$\sigma_e$is the electric conductivity of the fluid,μ is the viscosity and s_p is the specific heat of the fluid. The elastic parameter$\kappa < 0$for the second grade fluid [14], E is the heat generation/absorption, T_V is the temperature at any time and T_{ab} is the absolute temperature which is consider as constant. The conservation equations for mass and momentum, governing the flow and heat transfer process, can be stated as under the standard boundary layer assumptions are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{d_\rho s_p}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) - \frac{\sigma_e B_0^2 u}{d_\rho} - \kappa\left[u\frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial^2 u}{\partial x \partial y}\frac{\partial u}{\partial x} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right] \qquad (2)$$

$$u\frac{\partial T_v}{\partial x} + v\frac{\partial T_v}{\partial y} = \frac{\sigma_t}{\rho s_p}\frac{\partial}{\partial y}\left(\frac{\partial T_v}{\partial y}\right) + \frac{E}{\rho s_p}(T_v - T_{ab}) \qquad (3)$$

The appropriate boundary conditions for velocity are given by:

$$u = u_l(x) = -bx,\ v = v_l(x), \quad \text{at } y = 0 \qquad (4)$$

$$u \to 0, \frac{\partial u}{\partial y} \to 0 \text{ as } y \to \infty \qquad (5)$$

and the appropriate boundary conditions for temperature are given by

$$T_v = T_l(x) = T_{\mathrm{ab}} + C(x)^m \text{ at } y = 0 \qquad (6)$$

$$T_v \to T_{\mathrm{ab}} \text{ as } y \to \infty \qquad (7)$$

where b is a constant rate of linear stretching or shrinking (for shrinking b > 0 and for stretching b < 0), B is constant, T_l is the wall temperature.For m = 0 thermal boundary conditions become isothermal.

The stream function ψ satisfied the equation of continuity. The dimensionless stream function g(η) is defined as[15]

$$g(\eta) = \psi / x\sqrt{b\nu}. , \qquad \eta = y\sqrt{\frac{b}{\nu}} \qquad (8)$$

where η is the similarity variable.

$$\theta(\eta) = \frac{T_v - T_{ab}}{T_l - T_{\mathrm{ab}}} \qquad (9)$$

Using equations (8) and (9) in equation (2) and (3) we get Momentum and energy equations as,

$$g''' - (g')^2 + g.g'' = Bg' + \alpha[2g'.g''' - gg'''' - g''.g''.] \qquad (10)$$

$$\theta'' + Pd.g.\theta' - m.P_d.g'.\theta + Pd\,\lambda\,\theta = 0 \qquad (11)$$

Here differentiation with respect to η is represented by prime, B=$\frac{\sigma_e B_0^2}{b d_\rho}$is the magnetic parameter, $\alpha = \frac{kb}{\nu}$is the viscoelastic parameter which is less than zero for second grade fluid, $\lambda = \frac{E}{d_\rho s_P}$is internal heat generation/absorption parameter and P_d is Prandtl number.

The velocity components and boundary conditions become,

$u = bxg'$ (12)

$v = -\sqrt{b\nu}g$ (13)

$g(0) = s' \quad ,g'(0) = -1 \quad ,\theta(0) = 1$ (14)

$g'(\infty) \to 0\,, g''(\infty) \to 0, ,\theta(0) \to 0$ (15)

where $s' = \frac{-v_l}{\sqrt{b\nu}}$ is the mass transfer parameter. Using the above boundary conditions in momentum equation and heat transfer equation, we obtain the analytical solution of these equations in the form

$g(\eta) = s' - \frac{(1-e^{-\beta\eta})}{\beta}$ (16)

where is a real positive root of algebraic equation

$\alpha s'\beta^3 - \beta^2(1+\alpha) - s'\beta + B - 1 = 0$ (17)

Equation (16) provides the velocity profile, which is $g'(\eta) = -e^{-\beta\eta}$.Skin friction coefficient and the local Nusselt number, which are proportional to the numbers $g''(0)$and $-\theta'(0)$ respectively, are the major physical quantities of relevance. The parameter for skin friction is $g''(0)$=β.It should be mentioned that Cortell [4] examined the relationship between the skin friction parameter and the entrainment velocity $g(0)$ as function of the mass transfer parameter s' quantitatively for the two-dimensional shrinking described here. It is evident from the generic solution form (16) that when exponential solutions of the type (10) exist, the skin friction parameter and entrainment velocity parameter are connected as $g(0) = s' - \frac{1}{\beta} = s' - \frac{1}{g''(0)}$for the flow of a second-grade fluid over a shrinking sheet.

Numerical procedure

Using the Runge- Kutta method and the shooting approach, the set of equations (10) and (11) under the boundary conditions (14) and (15)can be numerically solved. Let

g=Y_1, g'=Y_2, g''=Y_3, g'''=Y_4,θ=Y_5,θ'=Y_6 (18)

Y'_1=Y_2,Y'_2=Y_3, Y'_3=Y_4, (19)

Y'_4=$\frac{1}{\alpha Y_1}(-Y_4 - Y_1Y_3 + Y_2^2 + BY_2 + 2\alpha Y_2Y_4 - \alpha Y_3^2)$ (20)

Y'_5=Y_6 (21)

$y_6' = -P_dY_1y_6 + mP_dY_2y_5 - P_d\lambda Y_5$ (22)

With the following initial conditions,

$Y_1(0)$ =s′,$Y_2(0)$=-1, $Y_3(0)$=w_1 , $Y_4(0)$=w_2 , $Y_5(0)$=1 , $Y_6(0)$=w_3 (23)

We only have two initial conditions, $g(0), g'(0)$ on $g(\eta)$, and one initial condition,$\theta(0)$ on $\theta(\eta)$ however we need six initial conditions to solve this system of equations. The unexplained beginning conditions $Y_3(0)$, $Y_4(0)$, and $Y_6(0)$ in equation (23) are assumed to be w_1, w_2, and w_3, respectively, in the shooting technique. Equation (20) is then numerically integrated as an initial valued problem with value set to η_∞. The correctness of the a priori anticipated missing starting condition is then validated by comparing the dependent variable's computed value at η_∞ to its provided value there.

If there is a difference, the procedure must be repeated while obtaining a better value for the missing beginning conditions. For convergence criterion of the order 10^{-7}, a step size of $\Delta\eta = 0.001$ was determined to be sufficient. A written Programme using the symbolic and computational computer language MATLAB performed the computations. We may thus derive numerical conclusions in regimes where the flow does not have perfect solutions of the form (16). We are able to fully categories the solutions to the heat transfer issue using the numerical solutions given here.

Result and discussion

To obtain numerical solutions for the governing ordinary differential equations of momentum and heat transfer (equations 10 and 11), the widely acclaimed Runge-Kutta method in conjunction with the shooting technique is employed. The boundary conditions (equations 14 and 15) are applied to solve the aforementioned equations. The figures presented (Figs. 1 through 9) offer visual representations of the relationship between various factors, including the shear stress at the wall ($g''(0)$), velocity ($g'(\eta)$), temperature field ($\theta(\eta)$), and the local rate of heat transfer at the surface or local Nusselt number ($-\theta'(0)$). These relationships are explored in relation to parameters such as the magnetic parameter (B), mass transfer parameter (s'), Prandtl number (P_d), viscoelastic parameter (α), variable wall temperature parameter (m), and internal heat generation/absorption parameter (E).

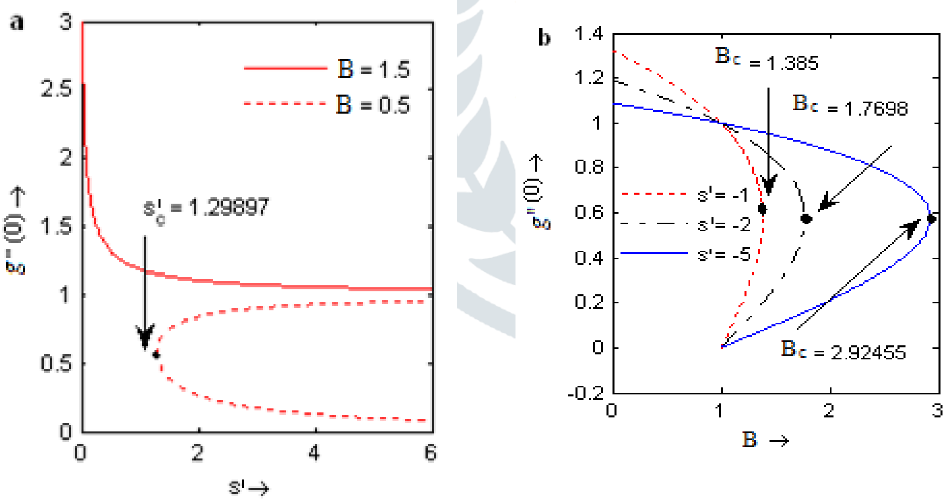

Fig. 1.s' (Mass suction parameter) vs. the g'' (0)(Skin friction coefficient) when α = -1.

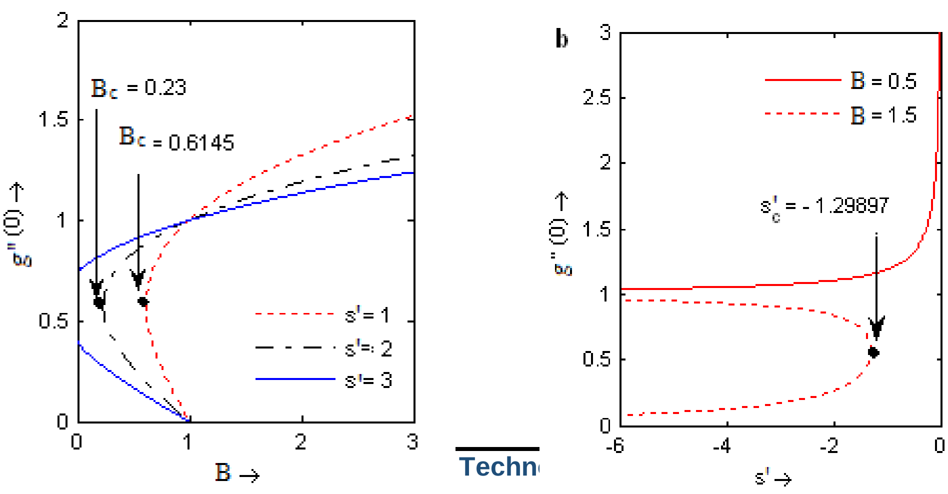

Fig. 2.B (Magnetic parameter) vs. the $g''(0)$ (Skin friction coefficient)when $\alpha = -1$.

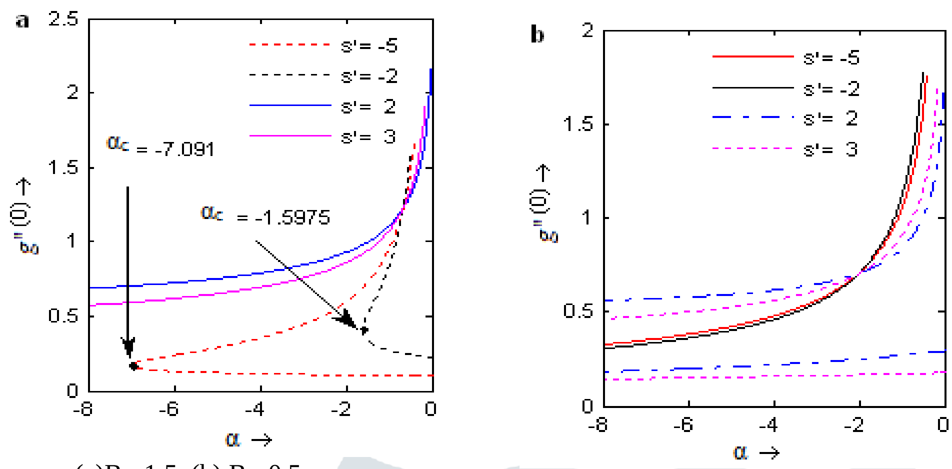

Fig. 3.α (Viscoelastic parameter)vs. the $g''(0)$(Skin friction coefficient)when (a)B= 1.5, (b) B= 0.5.

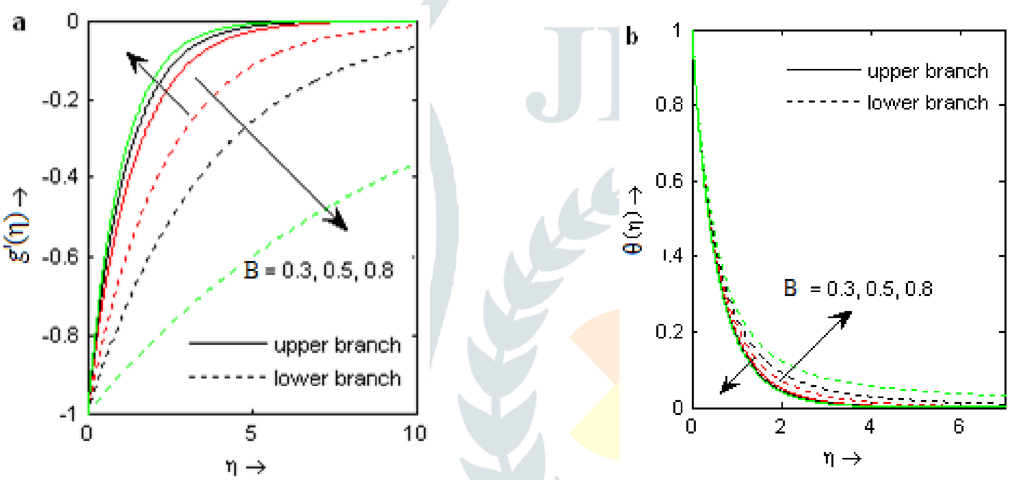

Fig. 4. (a) $g'(\eta)$(Dual velocity profiles) and (b) $\theta(\eta)$ (Dual temperature profiles) for different values of B(magmatic parameter) with $\alpha = -1$, m = 0, $s' = 2$, E = -0.5, $P_d = 1$

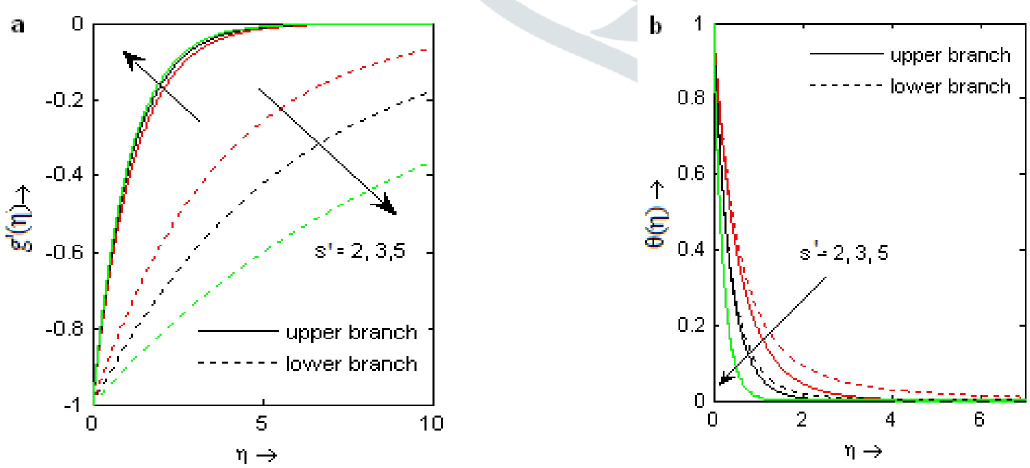

Fig. 5. (a) $g'(\eta)$(Dual velocity profiles) and (b) $\theta(\eta)$ (Dual temperature profiles) for different values of s' (mass suction parameter) with B= 0.5, m= 0,α= -1, $P_d = 1$, E = -0.5

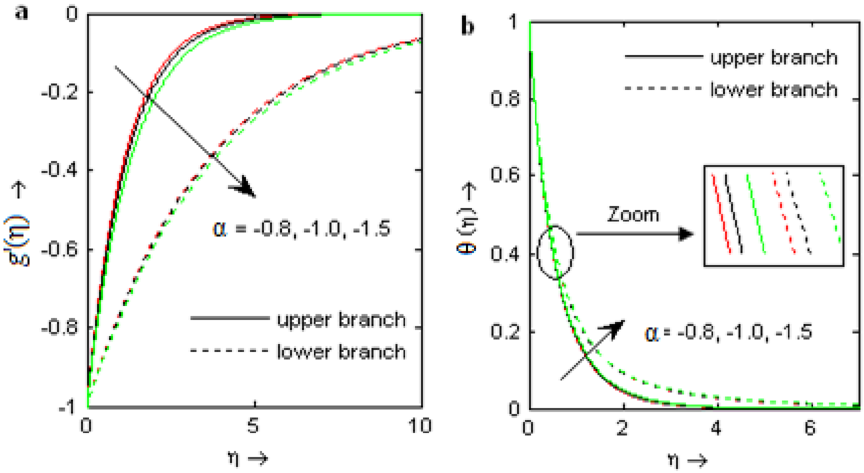

Fig. 6. (a) $g'(\eta)$ (Dual velocity profiles) and (b) $\theta(\eta)$ (Dual temperature profiles)for different values with $s' = 2$, m = 0, $P_d = 1$,B= 0.5, E = -0.5

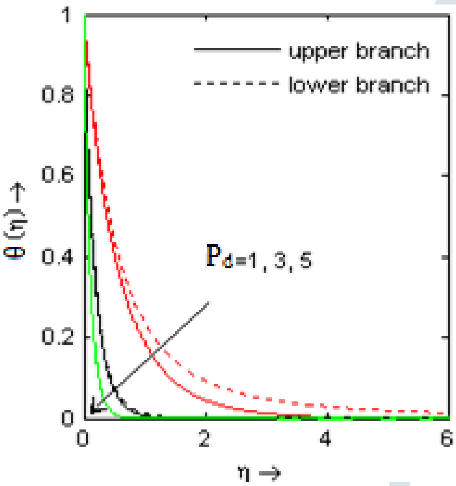

Fig.7. $\theta(\eta)$ (Dual temperature profiles) fordifferent values of P_d with $s' = 2$,$\alpha = -1$, m = 0,B= 0.5, E= -0.5

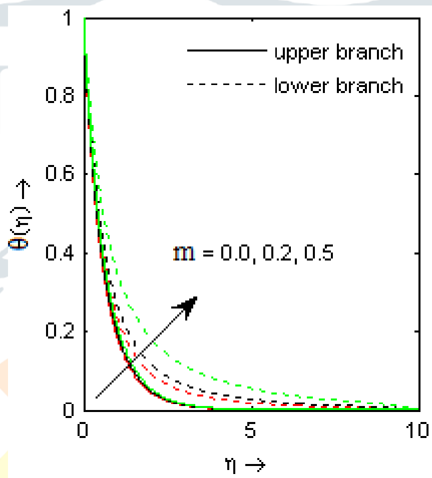

Fig.8. $\theta(\eta)$ (Dual temperature profiles) for different values of m with $s'= 2$,$\alpha = -1$, $P_d= 1$, B = 0.5, E = -0.5

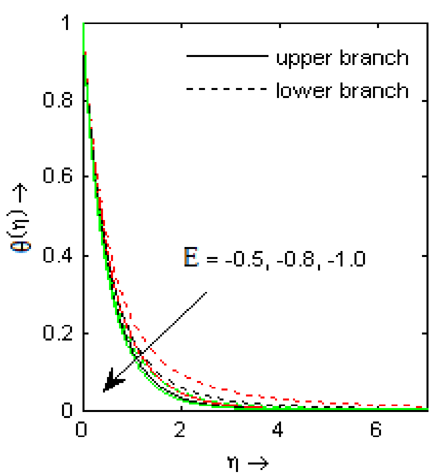

Fig. 9.$\theta(\eta)$(Dual temperature profiles)for different values of E with $s'= 2$, B= 0.5, $\alpha= -1$, m = 0, $P_d = 1$

Conclusions

The study yields the following conclusions:

1. The first solution exhibits a thinner thickness of the velocity boundary layer compared to the second solution, resulting in a corresponding decrease in the thickness of the thermal boundary layer.
2. Dual solutions have been observed for both the velocity field and temperature fields under certain parameter values.
3. In the context of the examined scenario, an interesting observation arises: when the magnitude of the $|\alpha|$, the parameter of visco-elasticity increases, both the thickness of the velocity boundary layer and the thermal boundary layer simultaneously expand for the upper as well as for the lower branch solutions.
4. The thermal boundary layer thickness is positively influenced by an increase in the wall temperature parameter r.
5. The thermal boundary layer thickness increases when the value of the wall temperature parameter (r) increases.

I believe that this endeavor will act as a catalyst for forthcoming experimental research.

References

[1] Sakiadis, B. C., Boundary Layer Behaviour on Continuous Solid Surface: I – Boundary Layer Equations for Two Dimensional and Axisymmetric Flows, AIChE. J., 7 (1961), 1, pp. 26-28

[2] Sakiadis, B. C., Boundary Layer Behaviour on Continuous Solid Surface: II – Boundary Layer on a Continuous Flat Surface, AIChE. J., 7 (1961), 2, pp. 221-225

[3] Singh, Vijendra, and Shweta Agarwal. "Heat transfer for two types of viscoelastic fluid over an exponentially stretching sheet with variable thermal conductivity and radiation in porous medium." Thermal Science 18, no. 4 (2014): 1079-1093.

[4] R. Cortell, On a certain boundary value problem arising inshrinking sheet flows, Appl. Math. and Comput. 217 (2010)4086-4093.

[5] M. Miklavcic, C.Y. Wang, Viscous flow due to a shrinkingsheet, Quart Appl Math 64(2) (2006) 283-290.

[6] T. Fang, J. Zhang, Closed-form exact solutions of MHD viscous flow over a shrinking sheet, Commun Nonlinear Sci Numer Simulat 14 (2009) 2853–2857.

[7] T. Fang, Boundary layer flow over a shrinking sheet withpower-law velocity, International Journal of Heat and Mass Transfer 51 (2008) 5838–5843.

[8] C.Y. Wang, Stagnation flow towards a shrinking sheet, International Journal of Non-Linear Mechanics 43 (2008) 377 –382.

[9] K. Bhattacharyya, Dual solutions in boundary layer stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet, Int. Commun. Heat Mass Transfer 38 (2011) 917-922.

[10] T. Hayat, S. Iram, T. Javed, S. Asghar, Shrinking flow of second grade fluid in a rotating frame: an analytic solution. Communications in Nonlinear Science and Numerical Simulation 15 (2010) 2932-2941.

[11] S. Nadeem, N. Faraz, Thin film flow of a second grade fluidover a stretching/shrinking sheet with variable temperature dependent viscosity. Chinese Physics Letters, 27(3) (2010)34704-34707.

[12] R.A. Van Gorder, K. Vajravelu, Multiple solutions for hydromagnetic flow of a second grade fluid over a stretching or shrinking sheet, Quart Appl Math 69 (2011) 405-424.

[13] Ghadhban, Fatimah Noaman, and Hayder Mohammad Jaffal. "Numerical investigation on heat transfer and fluid flow in a multi-minichannel heat sink: Effect of channel configurations." Results in Engineering 17 (2023): 100839.

[14] K. Vajravelu, D. Rollins, Hydromagnetic flow of a second grade fluid over a stretching sheet, Applied Mathematics and Computation 148 (2004) 783–791.

[15] C.Y. Wang, Liquid film on an unsteady stretching sheet. Quart. Appl. Math. 48 (1990) 601-610.